Electroweak two-loop contribution to the mass splitting within a new heavy $SU(2)_L$ fermion multiplet

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Abstract

New heavy particles in an $SU(2)_L$ multiplet, sometimes introduced in extensions of the standard model, have highly degenerate tree-level mass M if their couplings to the Higgs bosons are very small or forbidden. However, loop corrections may generate the gauge-symmetry-breaking mass splitting within the multiplet, which does not vanish in the large M limit due to the threshold singularity. We calculate the electroweak contribution to the mass splitting for a heavy fermion multiplet, to the two-loop order. Numerically, two-loop electroweak contributions are typically O(MeV).

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1 Introduction

In some extensions of the standard model, there are new heavy particles which belong to an $SU(2)_L \times U(1)$ multiplet F and have no, or very small, mixing with other particles. The masses of these particles are almost degenerate to a value M by the gauge symmetry. Although the spontaneous breaking of the $SU(2)_L \times U(1)$ symmetry by Higgs bosons may generate mass splitting δM among them, the tree-level mass splitting generally behaves as $\delta M \sim m_W^2/M$ and becomes very small for $M \gg m_W$. This is especially the case for a very heavy fermion multiplet where tree-level renormalizable couplings to the Higgs bosons $\bar{F}FH$ are forbidden by symmetry. Some of the examples are the almost pure winos or higgsinos [1, 2, 3, 4, 5, 6, 7, 8] in special parameter regions of the minimal supersymmetric standard model [9], $SU(2)_L$ triplet fermions in Type III seesaw model for neutrino masses [10, 11], and also models [12, 13] where vector-like heavy fermion multiplets are added to the standard model by hand.

In such cases, it has been known [2, 4, 6, 8, 11, 12, 13, 14] that the dominant part of the gauge-symmetry-breaking mass splitting within the multiplet F comes from the radiative correction. Although the form of the mass correction strongly depends on models, the contributions involving electroweak gauge bosons $V = (\gamma, Z, W)$, shown in Fig. 1(a) for the one-loop, are common in a wide class of extended models. Since gauge symmetry breaking in this diagram comes from the squared masses (m_W^2, m_Z^2) in the loops, one naively expect the $O(\alpha_2 m_W^2/M)$ contribution to the mass splitting. However, due to the singularity of the diagram near the threshold, at $p^2 = M^2 \sim (M + m_V)^2$, $O(\alpha_2 m_W)$ contribution to the mass splitting appears, which does not vanish in the $M \gg m_W$ limit: Roughly speaking, it is "nondecoupling". This mass splitting is phenomenologically interesting, especially in the case where the neutral component f^0 of F, either fermion or boson, is stable or has very long lifetime, and may be a candidate for the cosmological dark matter. In such a case, the loop-generated mass splitting between charged components $f^Q(Q \neq 0)$ of F and f^0 is crucial for estimating the rates of the $f^Q \to f^0 + \cdots$ decays expected at colliders, and also for possible resonant annihilation $f^0 f^0 \to f^Q f^{-Q} \to VV$ for indirect detection of f^0 [8, 13].

To evaluate the mass splitting within F to the next-to-leading order, we need two-loop calculation of the mass correction for the members of F. In this paper, we perform such calculation for the loop corrections by the standard model particles, generated by the electroweak gauge interactions of F. For simplicity, we concentrate on the $SU(2)_L$-breaking and "nondecoupling" part of the mass correction, which should be relevant for the mass splitting in the $M \gg m_W$ case.

2 One-loop mass correction

Since the electroweak contributions to the mass correction should be determined by the $SU(2)_L \times U(1)$ representation of F, we work in the framework of the Minimal Dark Matter

model [13], which has been proposed as a minimalist approach to the dark matter problem, for the fermion case. In this case, Dirac or Majorana fermions in an $SU(2)_L$ multiplet F with $SU(2)_L$ isospin I and $U(1)$ hypercharge Y (and having no $SU(3)$ color) are added to the standard model. The lagrangian is

$$\mathcal{L} = \mathcal{L}_{\rm SM} + c\bar{F}[\gamma^\mu D_\mu - M]F, \tag{1}$$

where $c = 1(1/2)$ for Dirac(Majorana) fermions, respectively. Note that the mass corrections presented in this paper are common to both types of fermions. D_μ denotes $SU(2)_L\times U(1)$ gauge covariant derivative for F. Since F has no direct couplings to the Higgs boson, the members of F, f^Q (with charge $Q = I_3 + Y$, $I_3 = -I, -I+1, \ldots, I$) have a common mass M at the tree-level. We assume that M is sufficiently larger than the masses of standard model particles (W, Z, top quark t, Higgs boson h), typically $M = O({\rm TeV})$ which is cosmologically favored in the Minimal Dark Matter model [13]. We also use approximation that all other particles in the standard model are massless.

The pole mass M_p of f^Q at the two-loop order is given in terms of the self energy of f^Q

$$\Sigma(p) \equiv \Sigma_K(p^2)\not{p} + \Sigma_M(p^2), \tag{2}$$

as

$$
\begin{aligned}
M_p &= \frac{M - \Sigma_M(M_p^2)}{1 + \Sigma_K(M_p^2)} \\
&= M - [M\Sigma_K^{(1)}(M^2) + \Sigma_M^{(1)}(M^2)] - [M\Sigma_K^{(2)}(M^2) + \Sigma_M^{(2)}(M^2)] \\
&\quad + [M\Sigma_K^{(1)}(M^2) + \Sigma_M^{(1)}(M^2)][\Sigma_K^{(1)}(M^2) + 2M^2\dot{\Sigma}_K^{(1)}(M^2) + 2M\dot{\Sigma}_M^{(1)}(M^2)] \\
&\equiv M + \delta M^{(1)} + \delta M^{(2)}.
\end{aligned} \tag{3}
$$

Here $\Sigma_{K,M}^{(1)}$ and $\Sigma_{K,M}^{(2)}$ are the one-loop and two-loop parts, respectively. The dot in Eq. (3) denotes the derivative with respect to the external momentum squared. The absorptive part of the self energy is $O(g^6)$ and need not be considered here. Loop integrals are regularized by the dimensional regularization ($D = 4 - 2\epsilon$) with the $\overline{\rm MS}$ subtraction scheme.

The form of the one-loop mass correction $\delta M^{(1)}$ is well known [2, 6, 8, 11, 12, 13, 14, 15]. Abbreviating the factor $\alpha_2/(4\pi)$, it is expressed as

$$\delta M^{(1)} = (C_F - I_3^2)X_W^{(1)} + s_W^2(I_3 + Y)^2 X_\gamma^{(1)} + c_W^2(I_3 - t_W^2 Y)^2 X_Z^{(1)}, \tag{4}$$

where $C_F = I(I+1)$, $c_W \equiv \cos\theta_W = m_W/m_Z$, $s_W \equiv \sin\theta_W$, $t_W \equiv \tan\theta_W$, and

$$
\begin{aligned}
X_V^{(1)} &= M\left[\left(2 + \frac{m_V^2}{M^2}\right)B_0(M^2, M, m_V) - 1 + \frac{1}{M^2}\{A(M) - A(m_V)\}\right] \\
&= M\left[\frac{3}{\epsilon} - 3\log M^2 + 4 - f\left(\frac{m_V}{M}\right)\right],
\end{aligned}
$$

3

$$f(x) \equiv 2x(2+x^2)\sqrt{4-x^2}\tan^{-1}\frac{\sqrt{2-x}}{\sqrt{2+x}} - x^2 + x^4\log x$$

$$= 2\pi x - 3x^2 + \frac{3}{4}\pi x^3 + O(x^4). \tag{5}$$

We use the Passarino-Veltman one-loop functions [16] defined as

$$A(m) = \frac{1}{\epsilon(1-\epsilon)}(m^2)^{1-\epsilon},$$

$$B_0(p^2, m_1, m_2) = \frac{1}{\epsilon}\int_0^1 dz\,[(1-z)m_1^2 + zm_2^2 - z(1-z)p^2 - i\delta]^{-\epsilon},$$

$$B_{22}(p^2, m_1, m_2) = \frac{1}{2\epsilon(1-\epsilon)}\int_0^1 dz\,[(1-z)m_1^2 + zm_2^2 - z(1-z)p^2 - i\delta]^{1-\epsilon}, \tag{6}$$

and

$$\tilde{B}_{22}(p^2, m_1, m_2) = B_{22}(p^2, m_1, m_2) - \frac{1}{4}[A(m_1) + A(m_2)]. \tag{7}$$

The $O(m_V)$ term of Eq. (5) gives the nondecoupling mass splitting within the multiplet. For example, for $Y = 0$, the one-loop mass splitting between f^Q and the neutral component f^0 of F is written as [13], independent of I,

$$M(f^Q) - M(f^0) = Q^2\Delta M^{(1)}. \tag{8}$$

where, in the $M \gg m_W$ limit,

$$\Delta M^{(1)} = \frac{\alpha_2}{2}(m_W - c_W^2 m_Z) = (166.99 \pm 0.07)\text{MeV}. \tag{9}$$

The numerical value in Eq. (9) is obtained by using the pole masses $m_W = (80.398\pm0.025)$ GeV, $m_Z = 91.1876$ GeV, $\alpha_2 = \alpha(m_Z)/s_W^2 = \alpha(m_Z)/(1-m_W^2/m_Z^2)$, and the QED running coupling in the $\overline{\text{MS}}$ scheme $\alpha(m_Z) = (127.93 \pm 0.03)^{-1}$, cited from Ref. [17], as input parameters. Note that the value (9) should change by ~ 1 MeV depending on choices of the renormalization scheme for the input parameters.

3   Two-loop mass correction

We now calculate the two-loop mass correction $\delta M^{(2)}$ coming from diagrams shown in Fig. 1(b-e). We use Feynman gauge fixing for simplicity, although the final result should not depend on the gauge fixing method.

The contribution of the diagram Fig. 1(b) with the insertion of the one-loop self energy of the electroweak gauge boson, $\Pi_{\mu\nu}^{V_1 V_2}(k) = g_{\mu\nu}\Pi^{V_1 V_2}(k^2) + O(k_\mu k_\nu)$, is written as

$$\delta M^{(2,1)} = -(C_F - I_3^2)\Delta\Sigma_{WW} - s_W^2(I_3 + Y)^2\Delta\Sigma_{\gamma\gamma}$$
$$-2s_W c_W(I_3 + Y)(I_3 - t_W^2 Y)\Delta\Sigma_{\gamma Z} - c_W^2(I_3 - t_W^2 Y)^2\Delta\Sigma_{ZZ}, \tag{10}$$
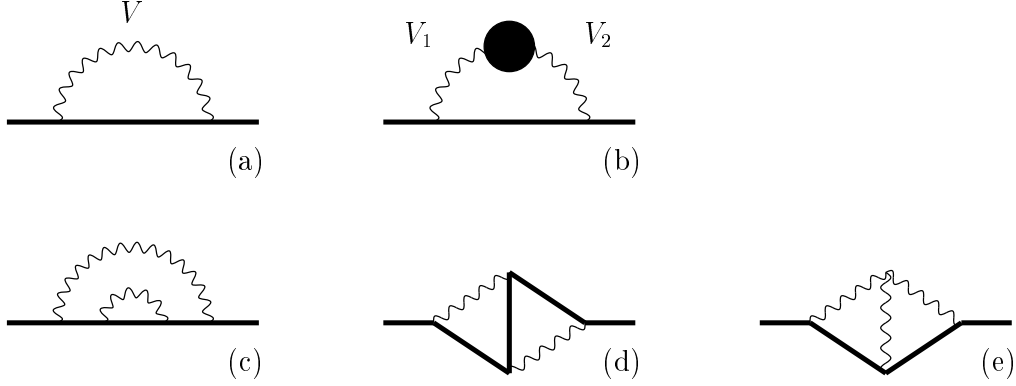
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Figure 1: One-loop (a) and two-loop (b-e) contributions to the self energy of the heavy fermions f in the multiplet F. The solid thick line and wavy line represent F and electroweak gauge bosons $V = (\gamma, Z, W)$, respectively. The black circle in (b) represents the one-loop self energy of the gauge bosons $\Pi^{V_1 V_2}$, by the standard model particles.

where

$$\Delta\Sigma_{V_1 V_2} = ig^2 \int \frac{d^D k}{(2\pi)^D} \frac{\gamma^\mu(\slashed{k} + \slashed{p} + M)\gamma_\mu \Pi^{V_1 V_2}(k^2)}{[k^2 - m_{V_1}^2][k^2 - m_{V_2}^2][(k+p)^2 - M^2]}\Bigg|_{\slashed{p}=M}. \tag{11}$$

Here we list the analytic forms of $\Pi^{V_1 V_2}(k^2)$ in the standard model for completeness [18]. The contributions from the (t, b) quark loops are, up to the overall factor $N_c \alpha_2/(4\pi)$ ($N_c = 3$ is the color number of quarks),

$$\Pi_{tb}^{WW}(k^2) = \frac{1}{2}\left[-4\tilde{B}_{22}(k_1^2, m_t, 0) - (k_1^2 - m_t^2)B_0(k_1^2, m_t, 0)\right], \tag{12}$$

$$\Pi_{tb}^{\gamma\gamma}(k^2) = \sum_{q=t,b} s_W^2 Q_q^2 \Pi_q^{vv}(k^2), \tag{13}$$

$$\Pi_{tb}^{\gamma Z}(k^2) = \sum_{q=t,b} \frac{s_W}{c_W} Q_q \left(\frac{1}{2}I_{3q} - Q_q s_W^2\right) \Pi_q^{vv}(k^2), \tag{14}$$

$$\Pi_{tb}^{ZZ}(k^2) = \sum_{q=t,b} \frac{1}{c_W^2}\left(\frac{1}{2}(I_{3q})^2 - I_{3q}s_W^2 Q_q + s_W^4 Q_q^2\right) \Pi_q^{vv}(k^2)$$

$$+ \frac{m_t^2}{2c_W^2}B_0(k^2, m_t, m_t), \tag{15}$$

where

$$\Pi_q^{vv}(k^2) \equiv -8\tilde{B}_{22}(k_1^2, m_q, m_q) - 2k_1^2 B_0(k_1^2, m_q, m_q). \tag{16}$$

The contributions of other quarks and leptons are obtained by appropriate changes of m_t, Q_q, and N_c. For the gauge and Higgs boson loops, we have, abbreviating the overall

factor $\alpha_2/(4\pi)$,

$$
\begin{aligned}
\Pi_{Vh}^{WW}(k^2) &= s_W^2[8(1-\epsilon)\tilde{B}_{22}(k^2, m_W, 0) + 4k^2 B_0(k^2, m_W, 0)] \\
&\quad + (1 + 8(1-\epsilon)c_W^2)\tilde{B}_{22}(k^2, m_W, m_Z) \\
&\quad + [4c_W^2 k^2 - m_Z^2 + 3m_W^2]B_0(k^2, m_W, m_Z) \\
&\quad + \tilde{B}_{22}(k^2, m_W, m_h) - m_W^2 B_0(k^2, m_W, m_h), \quad\quad (17) \\
\Pi_{Vh}^{\gamma\gamma}(k^2) &= s_W^2[4(3-2\epsilon)\tilde{B}_{22}(k^2, m_W, m_W) + 4k^2 B_0(k^2, m_W, m_W)], \quad\quad (18) \\
\Pi_{Vh}^{\gamma Z}(k^2) &= 4s_W c_W \left(3 - 2\epsilon - \frac{1}{2c_W^2}\right)\tilde{B}_{22}(k^2, m_W, m_W) \\
&\quad + s_W c_W (4k^2 + 2m_Z^2)B_0(k^2, m_W, m_W), \quad\quad (19) \\
\Pi_{Vh}^{ZZ}(k^2) &= \left[4c_W^2(3-2\epsilon) - 4 + \frac{1}{c_W^2}\right]\tilde{B}_{22}(k^2, m_W, m_W) \\
&\quad + [4c_W^2 k^2 + 4m_W^2 - 2m_Z^2]B_0(k^2, m_W, m_W) \\
&\quad + \frac{1}{c_W^2}[\tilde{B}_{22}(k^2, m_Z, m_h) - m_Z^2 B_0(k^2, m_Z, m_h)]. \quad\quad (20)
\end{aligned}
$$

In addition, there are also the contributions of F to $\Pi^{V_1 V_2}$. However, it is shown that the resulting $O(m_W)$ contributions to $\delta M^{(2)}$ are completely cancelled by the renormalization of the parameters in $\delta M^{(1)}$.

We may calculate the integrals (11) by extending the general formulas for the two-loop mass corrections [19], by including finite masses for (W, Z). However, since we are interested in the $SU(2)_L$-breaking and nondecoupling part of Eq. (11), it is prefered to expand the integrals (11) in $m_W(\sim m_Z, m_t, m_h)$ and then separate the $O(m_W)$ terms from the dominant and gauge-symmetric $O(M)$ terms, before numerical evaluation. This is achieved by applying the asymptotic expansion of the Feynman integrals near the threshold $p^2 = M^2$, as described in Ref. [20]. The $O(m_W)$ part of the integral (11) is then obtained as

$$
\Delta\Sigma_{V_1 V_2}|_{O(m_W)} \;\rightarrow\; ig^2 \int \frac{d^D k}{(2\pi)^D} \frac{2M}{[k^2 - m_{V_1}^2][k^2 - m_{V_2}^2](2k \cdot p)} \Pi^{V_1 V_2}(k^2). \quad\quad (21)
$$

In the following, we show only the $O(m_W)$ part (21) of the corrections $\Delta\Sigma_{V_1 V_2}$. By substituting the self energies (12–20), the integrals (21) are expressed in terms of the two-loop functions $(a = 1, 2)$

$$
\begin{aligned}
\frac{i}{(4\pi)^2} X_{0-a}(m_V, m_1, m_2) &\equiv \int \frac{d^D k}{(2\pi)^D} \frac{M}{[k^2 - m_V^2]^a (2k \cdot p)} \left[B_0(k^2, m_1, m_2) - \frac{1}{\epsilon}\right], \\
\frac{i}{(4\pi)^2} X_{22-a}(m_V, m_1, m_2) &\equiv \int \frac{d^D k}{(2\pi)^D} \frac{M}{[k^2 - m_V^2]^a (2k \cdot p)} \\
&\quad \times \left[B_{22}(k^2, m_1, m_2) - \frac{1}{\epsilon}\left(\frac{m_1^2 + m_2^2}{4} - \frac{k^2}{12}\right)\right], \quad\quad (22)
\end{aligned}
$$

6

and products of the one-loop functions. Note that the functions in Eq. (22) are independent of M and have no overall divergences. We calculate these functions by numerical integration of the Feynman parameter integrals shown below,

$$
X_{0-1}(m_V, m_1, m_2) = \pi m_V \left[\log \frac{m_V^2}{\mu^2} \right.
$$
$$
\left. - \int_0^1 dz \left(2 \frac{\sqrt{r_1(1-z) + r_2 z}}{\sqrt{z(1-z)}} - 2 \log(\sqrt{r_1(1-z) + r_2 z} + \sqrt{z(1-z)}) \right) \right],
$$
(23)

$$
X_{0-2}(m_V, m_1, m_2) = \frac{\pi}{2m_V} \left[\log \frac{m_V^2}{\mu^2} + 2 \int_0^1 dz \left(\frac{\sqrt{z(1-z)}}{\sqrt{r_1(1-z) + r_2 z} + \sqrt{z(1-z)}} \right. \right.
$$
$$
\left. \left. + \log(\sqrt{r_1(1-z) + r_2 z} + \sqrt{z(1-z)}) \right) \right],
$$
(24)

$$
X_{22-1}(m_V, m_1, m_2) = -\frac{\pi}{3} m_V^3 \left[\frac{1}{4} \{ 1 - 3(r_1 + r_2) \} \log \frac{m_V^2}{\mu^2} - \frac{2}{3} + \frac{9}{4}(r_1 + r_2) \right.
$$
$$
+ \int_0^1 dz \left(\{ -3z(1-z) + 2(1-z)r_1 + 2zr_2 \} \frac{\sqrt{(1-z)r_1 + zr_2}}{\sqrt{z(1-z)}} \right.
$$
$$
+ 3\{ z(1-z) - (1-z)r_1 - zr_2 \} \left\{ \log(\sqrt{r_1(1-z) + r_2 z} + \sqrt{z(1-z)}) \right.
$$
$$
\left. \left. \left. - \frac{1}{2} \log z(1-z) \right\} \right) \right],
$$
(25)

$$
X_{22-2}(m_V, m_1, m_2) = -\frac{\pi}{2} m_V \left[\frac{1}{4}(1 - r_1 - r_2) \log \frac{m_V^2}{\mu^2} - \frac{1}{2} + \frac{3}{4}(r_1 + r_2) \right.
$$
$$
+ \int_0^1 dz \left(-3\sqrt{z(1-z)} \sqrt{(1-z)r_1 + zr_2} + \{ 3z(1-z) - (1-z)r_1 - zr_2 \} \right.
$$
$$
\left. \left. \times \left\{ \log(\sqrt{(1-z)r_1 + zr_2} + \sqrt{z(1-z)}) - \frac{1}{2} \log(z(1-z)) \right\} \right) \right],
$$
(26)

where $r_{1,2} \equiv m_{1,2}^2 / m_V^2$. μ is the $\overline{\text{MS}}$ renormalization scale.

Here we show the explicit forms of the integrals (21), after subtracting $O(1/\epsilon)$ divergences from $\Pi_{V_1 V_2}$ by the $\overline{\text{MS}}$ scheme, and separating the mass corrections to the gauge bosons $\delta m_V^2 = -\text{Re}\Pi^{VV}(m_V^2)(V = W, Z)$ from $\Pi^{VV}(k^2)$. The (t, b) contributions coming from Eqs. (12–16) are, up to the overall factor $N_c \alpha_2 / (4\pi)$,

$$
\Delta \Sigma_{WW}^{tb} = \frac{1}{2} X_{WW}(m_t) - \frac{\pi}{m_W} \delta m_W^{2(tb)},
$$
(27)

$$
s_W^2 \Delta \Sigma_{\gamma\gamma}^{tb} = -4 s_W^4 Q_t^2 \pi^2 m_t,
$$
(28)

$$
2 s_W c_W \Delta \Sigma_{\gamma Z}^{tb} = 2 s_W^2 Q_t (I_{3t} - 2 Q_t s_W^2) X_{\gamma Z}(m_t)
$$
$$
+ 2 s_W^2 Q_b (I_{3b} - 2 Q_b s_W^2) X_{\gamma Z}(0),
$$
(29)

$$c_W^2 \Delta\Sigma_{ZZ}^{tb} = [(I_{3t})^2 - 2I_{3t}Q_t s_W^2 + 2Q_t^2 s_W^4]X_{ZZ}(m_t) - m_t^2 G_0(m_Z, m_t, m_t)$$
$$+[(I_{3b})^2 - 2I_{3b}Q_b s_W^2 + 2Q_b^2 s_W^4]X_{ZZ}(0) - c_W^2 \frac{\pi}{m_Z}\delta m_Z^{2(tb)}, \quad (30)$$

with

$$X_{WW}(m_t) = 8G_{22}(m_W, m_t, 0) + 2X_{0-1}(m_W, m_t, 0)$$
$$+2(m_W^2 - m_t^2)G_0(m_W, m_t, 0), \quad (31)$$

$$X_{\gamma Z}(m_t) = \frac{8}{m_Z^2}X_{22-1}(m_Z, m_q, m_q) + \frac{16\pi^2}{3}\frac{m_t^3}{m_Z^2}$$
$$+2X_{0-1}(m_Z, m_t, m_t) + \frac{4\pi m_t^2}{m_Z}\left(1 - \log\frac{m_t^2}{\mu^2}\right), \quad (32)$$

$$X_{\gamma Z}(0) = \pi m_Z \left(\frac{4}{3}\log\frac{m_Z^2}{\mu^2} - \frac{20}{9}\right), \quad (33)$$

$$X_{ZZ}(m_t) = 8G_{22}(m_Z, m_t, m_t)$$
$$+2X_{0-1}(m_Z, m_t, m_t) + 2m_Z^2 G_0(m_Z, m_t, m_t) \quad (34)$$

$$X_{ZZ}(0) = \pi m_Z \left(\frac{4}{3}\log\frac{m_Z^2}{\mu^2} - \frac{8}{9}\right). \quad (35)$$

Here we used the notations

$$G_{22}(m_V, m_1, m_2) \equiv X_{22-2}(m_V, m_1, m_2)$$
$$+\frac{\pi}{2m_V}[\text{Re}B_{22}(m_V, m_1, m_2) - ((m_1^2 + m_2^2)/4 - m_V^2/12)/\epsilon],$$
$$G_0(m_V, m_1, m_2) \equiv X_{0-2}(m_V, m_1, m_2) + \frac{\pi}{2m_V}[\text{Re}B_0(m_V, m_1, m_2) - 1/\epsilon], \quad (36)$$

and substituted analytic forms of the two-loop integrals (22) at $m_1 = m_2 = 0$ and at $m_V \to 0$. Analytic forms of other integrals involving m_t are shown in Appendix. Contributions of other quarks and leptons are obtained by taking $m_t \to 0$, where

$$X_{WW}(0) = \pi m_W \left(\frac{4}{3}\log\frac{m_W^2}{\mu^2} - \frac{8}{9}\right), \quad (37)$$

and, for leptons, changing (Q_q, N_c). Similarly, the gauge and Higgs boson contributions coming from Eqs. (17–20) are, up to the factor $\alpha_2/(4\pi)$,

$$\Delta\Sigma_{WW}^{Vh} = -16s_W^2 G_{22}(m_W, m_W, 0) - 8s_W^2(X_{0-1}(m_W, m_W, 0) + m_W^2 G_0(m_W^2, m_W, 0))$$
$$-2(8c_W^2 + 1)G_{22}(m_W, m_W, m_Z) + \frac{4}{3}\pi m_W - 8c_W^2 X_{0-1}(m_W, m_W, m_Z)$$
$$-2((5m_W^2 + m_Z^2)c_W^2 - m_Z^2 s_W^4)G_0(m_W, m_W, m_Z)$$
$$-2G_{22}(m_W, m_W, m_h) + 2m_W^2 G_0(m_W, m_W, m_h) - \frac{\pi}{m_W}\delta m_W^{2(Vh)}, \quad (38)$$

$$s_W^2 \Delta\Sigma_{\gamma\gamma}^{Vh} = 10\pi^2 s_W^4 m_W, \quad (39)$$

8

$$2s_W c_W \Delta\Sigma_{\gamma Z}^{Vh} = \frac{8s_W^2(6c_W^2 - 1)}{m_Z^2}[-X_{22-1}(m_Z, m_W, m_W) - \frac{2}{3}\pi^2 m_W^3 - \frac{\pi}{2}m_Z m_W^2(1 - \log\frac{m_W^2}{\mu^2})]$$
$$+ \frac{8}{3}s_W^2 c_W^2 \pi m_Z - 8s_W^2 c_W^2[3X_{0-1}(m_Z, m_W, m_W) + 2\pi^2 m_W], \tag{40}$$

$$c_W^2 \Delta\Sigma_{ZZ}^{Vh} = -2(12c_W^4 - 4c_W^2 + 1)G_{22}(m_Z, m_W, m_W) + \frac{4}{3}c_W^4 \pi m_Z$$
$$-8c_W^4 X_{0-1}(m_Z, m_W, m_W) - 4(4c_W^2 - 1)m_W^2[G_0(m_Z, m_W, m_W)]$$
$$-2G_{22}(m_Z, m_Z, m_h) + 2m_Z^2 G_0(m_Z, m_Z, m_h) - c_W^2 \frac{\pi}{m_Z}\delta m_Z^{2(Vh)}. \tag{41}$$

Other diagrams shown in Fig. 1(c–e) are also evaluated by using the threshold expansion [20], keeping only the $O(m_V)$ parts. Their sum, with subtracting subdivergences by the $\overline{\text{MS}}$ scheme and after the (one-loop)×(one-loop) term in Eq. (3) is added, is given as

$$\delta M^{(2,2)} = 4\pi m_W(C_F - I_3^2)\left[c_W^2 \log\frac{m_Z^2}{\mu^2} + (2 - c_W^2)\log\frac{m_W^2}{\mu^2} + 4s_W^2(-1 + \log 2)\right]$$
$$+ 8\pi c_W^2 m_Z I_3(I_3 - t_W^2 Y)\log\frac{m_W^2}{\mu^2} - 4\pi c_W^2(C_F - I_3^2)f_{ZW}. \tag{42}$$

Here

$$f_{ZW} \equiv -\frac{1}{3}(2 + c_W^2)m_W \int_0^1 dz\, z^{-3/2}(1 - z)^{-1/2}[(c_W^{-2}z + 1 - z)^{3/2} - 1]$$
$$-\frac{1}{3}(2 + c_W^{-2})m_Z \int_0^1 dz\, z^{-3/2}(1 - z)^{-1/2}[(c_W^2 z + 1 - z)^{3/2} - 1]$$
$$\sim -0.027m_W, \tag{43}$$

is the two-loop function appearing in Fig. 1(c,d) with both W and Z bosons.

We then need to add the counterterms coming from the renormalization of the parameters in the one-loop contributions (4, 5); (m_W, m_Z) in $X_{W,Z}^{(1)}$ and $(\alpha_2, c_W^2, \ldots)$ in the coupling constants. We adopt the scheme where the pole masses (m_W, m_Z) and the $\overline{\text{MS}}$ running coupling of QED $\alpha(m_Z)$, which are used in Eq. (9), are chosen as the input parameters. In this scheme, the renormalization is achieved by removing the last $O(\delta m_V^2)$ terms from $\Delta\Sigma_{WW}$ (27, 38) and $\Delta\Sigma_{ZZ}$ (30, 41), and adding the counterterms for $(\alpha_2, c_W^2, \ldots)$ expressed as tree-level functions of $(m_Z, m_W, \alpha(m_Z))$. It is checked that the final form of the two-loop $O(m_W)$ mass correction to f^Q is finite and independent of the $\overline{\text{MS}}$ renormalization scale μ.

Here we comment on the mass splitting of a new heavy scalar $SU(2)_L$ multiplet S. In contrast to the case of the fermion multiplet, direct couplings of S to the Higgs bosons, such as $S^*SH^\dagger H$, should always exist [13, 21]. Nevertheless, assuming that the effect of these direct couplings is negligible, we have verified that the nondecoupling $O(m_W)$ parts of the one-loop [13, 22] and two-loop mass corrections δM are identical to those for the fermions in the same gauge representation. This result is quite natural in the view that the $O(m_W)$ mass correction could be understood as the energy of the electroweak gauge fields around a static point source, and should be insensitive to the spin of the source particle [13].

4   Numerical results

We show the numerical results of the two-loop contributions to the mass splitting within the $Y = 0$ fermion multiplet. As seen in Eqs. (10, 42), the one-loop relation (8) still holds with the change $\Delta M^{(1)} \to \Delta M^{(1)} + \Delta M^{(2)}$, where $\Delta M^{(2)} = \Delta M^{(2,ql)} + \Delta M^{(2,Vh)}$.

The contribution $\Delta M^{(2,ql)}$ of the quark-lepton subloop diagrams (including corresponding counterterms) is shown in Fig. 2 as a function of m_t. At $m_t = 171$ GeV, there is cancellation between the (t, b) subloop contribution, shown in the dashed line, and remaining contribution with subloops of other quarks or leptons, -3.3 MeV, giving the total shift -1.5 MeV at $m_t = 171$ GeV.
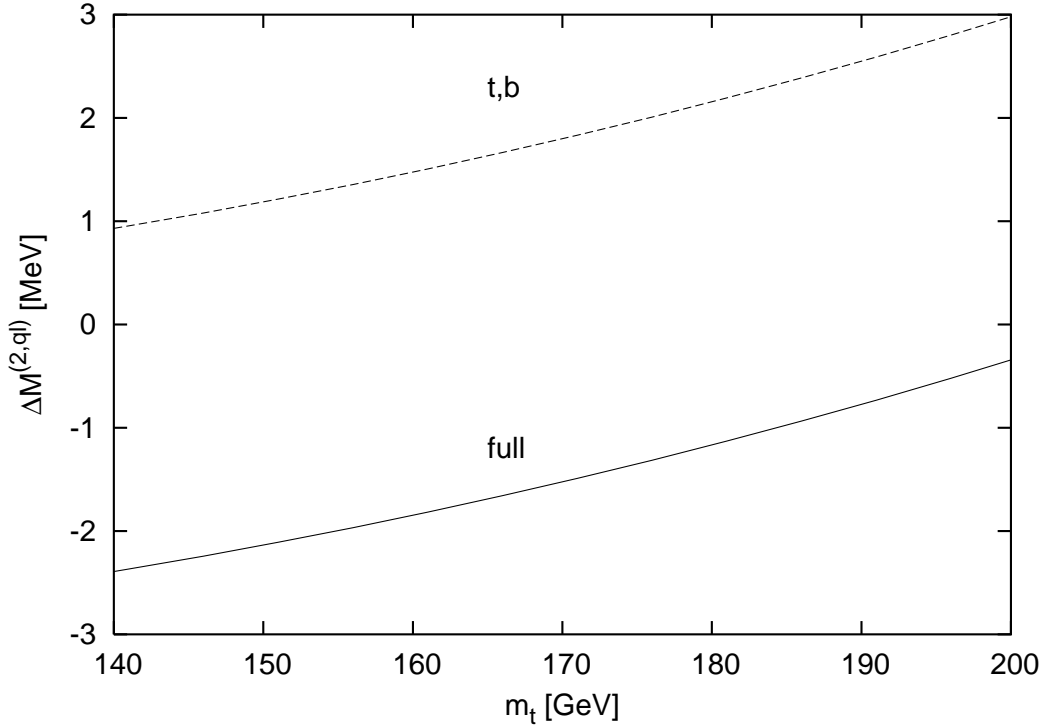

Figure 2:   Two-loop contribution to the mass splitting $\Delta M^{(2,ql)}$ between fermions in a heavy SU(2)$_L$ multiplet with $Y = 0$, from diagrams in Fig. 1(b) with quark and lepton subloops. Solid and dashed lines denote full and (t, b) subloop contributions, respectively.

The remaining contribution $\Delta M^{(2,Vh)}$ from diagrams without quarks or leptons (again including corresponding counterterms) is shown in Fig. 3 as a function of m_h. At $m_h = 140$ GeV, the shift is -0.9 MeV, smaller than the quark-lepton loops.

These two-loop contributions are much smaller than the $O(m_W)$ part of the leading one-loop contribution (4), as expected. However, for $Y = 0$, it may compete with the M-dependence of the one-loop contribution (8) which behaves like $-0.5(1\,\text{TeV}/M)^2$ MeV for large M due to the accidental cancellation of the $O(m_W^2/M)$ term in Eq. (8). In
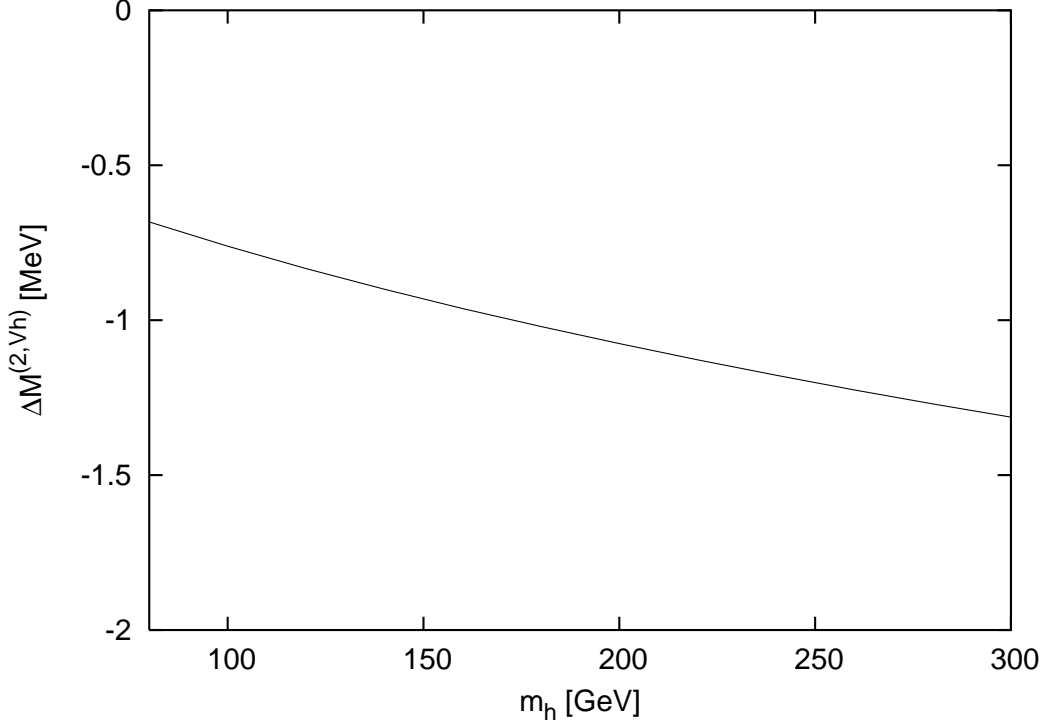
Figure 3: Two-loop contribution to the mass splitting $\Delta M^{(2,Vh)}$ between fermions in a heavy $SU(2)_L$ multiplet with $Y = 0$, from diagrams in Fig. 1(b-e) with gauge and Higgs bosons.

comparison, in the case of the higgsino-like doublet $F = (f^+, f^0)$ with ($I = 1/2$, $Y = 1/2$), the two-loop corrections to the mass splitting $M(f^+) - M(f^0)$, which is $\alpha m_Z/2 = 356.4$ MeV at the one-loop, is -1.2 MeV from quark and lepton loops at $m_t = 171$ GeV, and -1.8 MeV from gauge and Higgs boson loops at $m_h = 140$ GeV, respectively.

5   Conclusion

We have calculated the two-loop electroweak contribution to the $O(m_W)$ correction to the masses of new heavy fermions in an $SU(2)_L$ multiplet F, which causes gauge-symmetry-breaking and "nondecoupling" mass splitting within F. Analytic formula of the $O(m_W)$ mass corrections have been presented for F in general $SU(2)_L \times U(1)$ representation. The two-loop contribution has turned out to be typically O(MeV), which is of similar order to the M dependence of the one-loop contribution for the $Y = 0$ case.

Appendix

In the case of the (t, b) contributions (27–30), Feynman parameter integrals for the functions (22) can be analytically performed. For $m_1 = m_2 \equiv \sqrt{r} m_V$ with $r > 1/4$, we have

$$X_{0-1} = \pi m_V \left[\log \frac{m_V^2}{\mu^2} - 2 + \log r - 2\sqrt{4r-1} \tan^{-1} \sqrt{4r-1} \right], \tag{A.1}$$

$$X_{0-2} = \frac{\pi}{2m_V} \left[\log \frac{m_V^2}{\mu^2} + \log r + \frac{2}{\sqrt{4r-1}} \tan^{-1} \sqrt{4r-1} \right], \tag{A.2}$$

$$X_{22-1} = -\frac{\pi}{3} m_V^3 \left[\frac{1}{4}(1 - 6r) \log \frac{m_V^2}{\mu^2} - \frac{2}{3} + \frac{7}{2} r \right.$$
$$\left. + \frac{1-6r}{4} \log r + \frac{1}{2}(4r-1)^{3/2} \tan^{-1} \sqrt{4r-1} \right], \tag{A.3}$$

$$X_{22-2} = -\frac{\pi}{2} m_V \left[\frac{1}{4}(1 - 2r) \log \frac{m_V^2}{\mu^2} - \frac{1}{2} + \frac{1}{2} r \right.$$
$$\left. + \frac{1-2r}{4} \log r - \frac{1}{2}\sqrt{4r-1} \tan^{-1} \sqrt{4r-1} \right]. \tag{A.4}$$

For $m_2 = 0$ and $m_1 \equiv \sqrt{r} m_V$,

$$X_{0-1} = \pi m_V \left[\log \frac{m_V^2}{\mu^2} - 2 - 2\sqrt{r} + r \log r - 2(r-1) \log(1 + \sqrt{r}) \right], \tag{A.5}$$

$$X_{0-2} = \frac{\pi}{2m_V} \left[\log \frac{m_V^2}{\mu^2} - 2\sqrt{r} - r \log r + 2(r+1) \log(1 + \sqrt{r}) \right], \tag{A.6}$$

$$X_{22-1} = -\frac{\pi}{3} m_V^3 \left[\frac{1}{4}(1 - 3r) \log \frac{m_V^2}{\mu^2} + \frac{1}{12}(-8 - 6\sqrt{r} + 21r + 16r^{3/2} - 3r^2 + 6r^{5/2}) \right.$$
$$\left. + \frac{(r-3)r^2}{4} \log r - \frac{1}{2}(r-1)^3 \log(1 + \sqrt{r}) \right], \tag{A.7}$$

$$X_{22-2} = -\frac{\pi}{2} m_V \left[\frac{1}{4}(1 - r) \log \frac{m_V^2}{\mu^2} + \frac{1}{12}(-6 - 6\sqrt{r} + 3r + 4r^{3/2} + 3r^2 - 6r^{5/2}) \right.$$
$$\left. - \frac{r^2(r-1)}{4} \log r + \frac{1}{2}(r-1)(r^2-1) \log(1 + \sqrt{r}) \right]. \tag{A.8}$$

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